CR13: Computational Topology
Exercises #1

1. A graph embedded on the plane is a **triangulation** if every face is adjacent to exactly three edges. Show that if G is a triangulation, its numbers of vertices v and faces f satisfy $2v = 4 + f$.

2. Let G be a simple planar graph, and suppose we arbitrarily color each edge of G either blue or red. Prove that for any embedding of G in the plane, there exists a vertex around which the incident red edges are consecutive. *Hint: Add edges to G to obtain a triangulation G', then build a well-chosen bipartite graph out of G', look at the degrees of its vertices and apply the previous question.*

3. A graph is **directed** if every edge is endowed with an orientation from one vertex to the other one. A **source** is a vertex with only outgoing edges, and a **sink** is a vertex with only incoming edges. A directed graph is a **directed acyclic graph** (DAG) if it does not contain any directed cycle. Let G be a planar DAG with a unique source and a unique sink. Prove that in any planar embedding of G, for every vertex v of G, all the incoming edges are consecutive around v.