CR13: Computational Topology
Exercises #2

1. Let C be the 1-skeleton of a three dimensional cube, i.e., the graph formed by its vertices and edges. Prove that C is 3-connected.

   Let e be one of the four diagonals of the cube. Deduce from the preceding question that the graph $C+e$, obtained by adding edge e to C, is not planar.

   Give two other proofs of the nonplanarity of $C+e$.

2. Let G be a 3-connected graph with at least 6 vertices. Suppose that G contains a subdivision K of the complete graph K_5. Every edge of K_5 corresponds to a path in K which we call a **branch**.

   If K contains a vertex of degree two (in K), show that G contains a path whose interior is disjoint from K and that joins a vertex interior to some branch B in K to a vertex in another branch B'. This vertex may be an endpoint of B', but not an endpoint of B.

   Deduce that every 3-connected nonplanar graph with at least 6 vertices contains a subdivision of $K_{3,3}$.