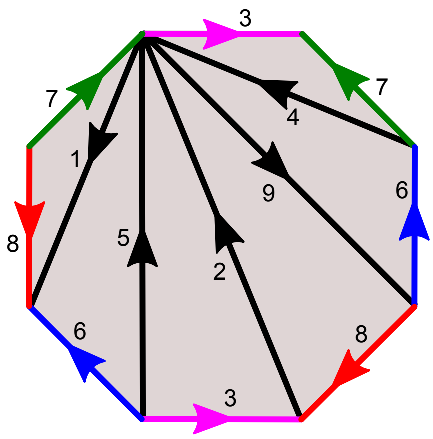
CR13: Computational Topology
Exercises #3

Recall that the genus of a graph is the minimal genus of all the orientable surfaces where the graph can be embedded. Its **non-orientable genus** is similarly defined as the minimal genus of all the *non-orientable* surfaces where the graph embedds.

1. What is the genus, respectively non-orientable genus, of each of the complete graphs K_4, K_5, K_6 and K_7? Justify your answers. (Beware that the proof for the non-orientable genus of K_7 is far from trivial.)

2. The next figure represents a triangulated polygon whose arcs, *i.e.*directed edges, are assigned values in $\mathbb{Z}_{19}$, the set of integers modulo 19. Two opposite arcs should receive opposite values, so that only one of the two orientations of an edge are shown.



   Suppose that the sides of this polygon are identified according to their labels and orientations. What is the genus of the resulting surface?

   Consider the collection of triplets $(i, j, k) \in \mathbb{Z}_{19} \times \mathbb{Z}_{19} \times \mathbb{Z}_{19}$ such that $j - i, k - j$ and $i - k$ are the labels of the arcs of some oriented triangle in the above figure. (Each triangle has two orientations, each inducing a coherent orientation of its edges.) Show that the collection of triplets (considered as unordered sets) defines a triangulation of some surface, where the elements of $\mathbb{Z}_{19}$ are the vertices of this triangulation and two triplets (i, j, k) and (ℓ, m, n) defines adjacent triangles if they share two elements, *i.e.*$|\{i, j, k\} \cap \{\ell, m, n\}| = 2$.

   Use the previous question to compute the genus of K_{19}.