# CR13: Computational Topology
# Exercises #5

1. Let $G$ be the group $< a, b \mid a^4, b^2a^{-2}, abab^{-1} >$. Show that $aba^3b = 1$, that $a$ and $b$ are distinct and non-trivial and that $a$ and $a^{-1}$ are conjugate.

2. Show that the groups $G =< x, y \mid x^3, y^2, (xy)^2 >$ and $G' =< y, z \mid (zy)^3, y^2, z^2 >$ are isomorphic. Can you recognize that group?

3. Let $G$ be the group $< x, y \mid x^5, y^3, yxy^{-1}x^{-3} >$. How many elements does $G$ have? *Hint: It's not 15.*

4. Let $T$ be a torus, and recall that its fundamental group is $\pi_1(T) = \mathbb{Z}^2$. A closed curve on the torus is **simple** if it has no self-intersections. Show that an element $(p, q) \in \pi_1(T)$ can be represented by a simple closed curve if and only if $p$ and $q$ are relatively prime. *Hint: Look at the curves in the universal cover of the torus.*