CR13: Computational Topology
Exercises #8

1. What familiar space is the Δ-complex obtained from the 2-simplex $[v_1, v_2, v_3]$ by identifying the edges $[v_1, v_2]$ and $[v_2, v_3]$, with these orientations?

2. Compute the homology groups of the triangular parachute obtained from identifying the three vertices of a 2-simplex to a single point.

3. Find a structure of Δ-complex describing the n-dimensional sphere $\mathbb{S}^n$. Use it to compute the homology groups of n-dimensional spheres, and to deduce that $\mathbb{S}^n$ and $\mathbb{S}^m$ are not homeomorphic[1] for $n \neq m$. Could we have told them apart using the fundamental group?

4. Compute the homology group of the Δ-complex X obtained from Δ^n by identifying all faces of the same dimension (with the orientations induced by that of Δ^n). Thus X has a single k-simplex for each $k \leq n$.

5. Take a single 3-simplex and label its vertices by $0, 1, 2$ and 3. Identify the $[0, 1, 2]$ face with the $[1, 3, 0]$ face by sending the vertices $0, 1$ and 2 respectively to $1, 3$ and 0. Compute the homology of the resulting space. Can you recognize that space? (*This is **very** tricky to visualize. The homology should give some indication. Finding the complex of question 1 inside that one might also help. One can also try to recognize the topology of its boundary.)*

[1] Of course, at its core this proof relies of Theorem 2.2 in the lectures notes, which we have not proved.