# Algorithmic Topology and Groups
# Exercises #4

1. Show that the Euler characteristic $\chi = |V| - |E| + |F|$ of an (oriented) map, where $V, E, F$ are its number of vertices, edges and faces, respectively, is even integer no larger than 2. Conclude that the genus of a map is a non-negative integer.

2. Consider the Cayley graph, $G_7$, of $\mathbb{Z}/7\mathbb{Z}$ with respect to the three generators $\{1,2,3\}$. Let $r = (1,3,2,-1,-3,-2)$ be a cyclic permutation of $\mathbb{Z}/7\mathbb{Z} \setminus \{0\}$. Compute the genus of the map whose underlying graph is $G_7$ and whose rotation system at each vertex is given by $r$.

3. The *genus* of a graph is the minimal genus of all the orientable surfaces where the graph can be embedded. What is the genus of the complete graph over $n$ vertices for $n = 4,5,6,7$? Justify your answers. (Hint: you may use the result of exercise 2 )

4. Show that the contraction of a pendant edge (an edge with a degree one endpoint) can be obtained from a sequence of edge and face subdivisions and their inverses.

5. Describe the maps $(A, \rho, \iota)$ for which $\rho$ is an automorphism. Same question with $\iota$ in place of $\rho$.