Algorithmic Topology and Groups
Exercises #5

1. Propose an algorithm to compute a basis of the fundamental group of a finite map. Analyse its complexity.

2. Suppose that the edges of a finite map M are positively weighted. Propose an algorithm to compute a minimal basis of M. Analyse its complexity.

3. With the assumptions of the previous exercise, propose an algorithm to compute a shortest non-contractible circuit in M. Analyse its complexity. (When all the edges have unit weight, the length of this shortest circuit is called the *edge-width* of M in graph theory. It is a combinatorial version of the systole in Riemannian geometry.)

4. Let M be a map and let $b : F(M) \to \{0,1\}$, be a *boundary indicator* defined over the faces of M. A face of M is said *perforated* if its boundary indicator is 1, and *plain* otherwise. We realize the pair (M,b) as a topological surface with boundary as follows. Thicken the graph $G(M)$ as for the topological realization of the map M and close each plain facial circuit of this thickening with a disk. Equivalently, one can realize (M,b) by gluing facial polygons and remove a small open disk in each polygon corresponding to a perforated face.

   We consider the homotopy relation for (M,b) generated by addition/removal of spurs and replacement of complementary paths of facial circuits of plain faces only.

   Generalize the construction of a quad system for such pairs (M,b), that allows to transform a path of M into a homotopic path of the quad system in linear time.